APPROXIMATING MINIMAL COMMUNICATED EVENT SETS FOR DECENTRALIZED SUPERVISORY CONTROL

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Abstract: This paper discusses the problem of selecting the minimal set of events to be communicated between decentralized supervisory controllers in order for the behavior of a controller system to match a given specification. This optimization problem is known to be computationally difficult, so this paper discusses the problem of approximating this set of communicated events. It is shown how the communication minimization problem is related to a centralized supervisory control minimal sensor selection problem and a special type of directed graph *st*-cut problem. Polynomial time algorithms to approximate solutions to these problems most likely do not exist (using worst-case analysis), but several effective heuristic approximation methods are shown for these problems that work well for average cases.

Keywords: Discrete-Event Systems, Supervisory Control, Decentralized Control Systems, Approximation Algorithms, Heuristics.

1. INTRODUCTION

Many large and complex systems are commonly most effectively controlled through the use of decentralized control methods due to the fact that centralized controllers may not be economically feasible to implement. Decentralized controllers make local (possibly unique) observations of system behavior and enforce local control actions in order for the global controlled system behavior to match a given specification. Unfortunately, for many system behavior specifications that can be achieved through the use of centralized control systems, decentralized controllers might not exist for systems to achieve the same specification. However, if decentralized controllers were allowed to communicate, the controllers would be able to achieve more specifications. In fact, decentralized controllers with unlimited communication are effectively centralized controllers. Unlimited communication between controllers may not always be feasible, so an interesting problem for a given decentralized control specification would be to

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find the minimal communication sufficient for the specification to be achievable.

It is hypothesized by van Schuppen (2004) that the general problem of synthesizing communicating decentralized controllers is likely undecidable. With this in mind, this paper discusses a special case of an open problem in the framework of decentralized supervisory control discussed by van Schuppen (2004) where the decentralized controllers are restricted to communicate all occurrences of a subset of their locally observed events. The problem is to find the minimal cardinality subsets of events to be communicated that are sufficient for the specification to be achievable.

The investigations in this paper are in the framework of decentralized supervisory control as introduced by Rudie and Wonham (1992). The work in this paper considers only the two controller case, but the results contained herein can be easily generalized. A variation of this problem is also discussed by Wong and van Schuppen (1996) where asymmetric communication is assumed. Due to the necessary brevity of this paper, formal proofs are not shown.

In the next section, the minimal communication decentralized control problem is formulated in the framework of Rudie and Wonham (1992). Section 3 relates the minimal communication problem to a graph cutting problem and a centralized control sensor selection problem. Heuristic methods are shown in Section 4 to calculate approximate solutions to the minimal communication problem. Section 5 closes the paper with a brief discussion.

2. THE COMMUNICATION SELECTION PROBLEM

In this paper systems and specifications are modeled as the automata $G = (X^G, x_0^G, \Sigma, \delta^G)$ and $H = (X^H, x_0^H, \Sigma, \delta^H)$, respectively. The behavior generated by G is denoted by $\mathcal{L}(G)$ and the behavior generated by the decentralized controllers S_1 and S_2 controlling G is denoted by $\mathcal{L}(S_1 \wedge S_2/G)$ (assuming conjunctive decentralized control as in Rudie and Wonham (1992)). The system $S_1 \wedge S_2/G$ is said to match the specification H if $\mathcal{L}(S_1 \wedge S_2/G) = \mathcal{L}(H)$.

The local controllable events of controller S_i ($\Sigma_{ci} \subseteq \Sigma$) and the local observable events of controller S_i ($\Sigma_{oi} \subseteq \Sigma$) are those events that can be respectively disabled or observed by controller S_i. Due to the controllability and co-observability theorem from Rudie and Wonham (1992), there exists conjunctive controllers S_1 and S_2 such that $\mathcal{L}(S_1 \wedge S_2/G) = \mathcal{L}(H)$ if and only if $\mathcal{L}(H)$ is controllable with respect to $\mathcal{L}(G)$ and $\Sigma \setminus (\Sigma_{c1} \cup \Sigma_{c2})$ and $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}. See Cassandras and Lafortune (1999) for a deeper introduction to supervisory control. It is assumed without loss of generality that $\mathcal{L}(H)$ is always controllable.

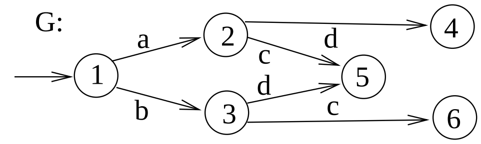

Fig. 1. System G for Example 2.

It might not always be true that $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}. This occurs when decentralized controllers do not have sufficient information to make appropriate control decisions about the system due to their observations alone. This deficiency could be overcome if the controllers are allowed to communicate. Let $\Sigma_{oij} \subseteq \Sigma_{oi}$ be the set of events that when observed by controller i are immediately communicated to controller j. This communication protocol effectively makes the events Σ_{oij} observable to controller j. With this in mind, if $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, $\Sigma_{o1} \cup \Sigma_{o21}$, $\Sigma_{o2} \cup \Sigma_{o12}$, Σ_{c1} and Σ_{c2}, then the pair $(\Sigma_{o12}, \Sigma_{o21})$ is called a *sufficient communication selection* because the communication of the events $(\Sigma_{o12}, \Sigma_{o21})$ gives the decentralized controllers sufficient local information about system behavior to achieve the specification.

Also without loss of generality, it is assumed that $\mathcal{L}(H)$ is always co-observable with respect to $\mathcal{L}(G)$, $\Sigma_{o1} \cup \Sigma_{o2}$, $\Sigma_{o2} \cup \Sigma_{o1}$, Σ_{c1} and Σ_{c2}. That is, a control objective can be achieved if all event observations are communicated. Unfortunately, due to reasons of economy or simplicity, it may be desired that as few events as possible are selected to be communicated. That is, the cardinality of $\Sigma_{o12} \cup \Sigma_{o21}$ should be as small as possible. This problem of finding such a minimal cardinality sufficient communication selection is known as the *communication selection problem*.

Problem 1. Communication Selection: Given G, H, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}, find a sufficient communication selection (Σ_{o12}^{min}, Σ_{o21}^{min}) such that for any other sufficient communication selection $(\Sigma_{o12}, \Sigma_{o21})$, $|\Sigma_{o12}^{min} \cup \Sigma_{o21}^{min}| \leq |\Sigma_{o12} \cup \Sigma_{o21}|$.

Example 2. An example of Problem 1 is now given. Consider the system G in Figure 1 with $\Sigma = \{a, b, c, d\}$ and H is a copy of G such that states 4 and 6 are removed. Let $\Sigma_{o1} = \Sigma$, $\Sigma_{c1} = \emptyset$, $\Sigma_{o2} = \emptyset$ and $\Sigma_{c1} = \Sigma$. Note that controllers cannot be synthesized to achieve the given specification unless the controllers allowed to communicate. This is because Controller 1 has insufficient actuation to perform the correct control action while Controller 2 has insufficient information. For this problem *the minimal sufficient communication selection pair is* $(\{a,b\},\emptyset)$.

Problem 1 is NP-complete due to a polynomial-time many-one reduction from the centralized sensor selection problem (Rohloff and van Schuppen). However, it may still be required to find a communication selection $(\Sigma_{o12},\Sigma_{o21})$ such that $|\Sigma_{o12}\cup\Sigma_{o21}|$ is approximately $|\Sigma_{o12}^{min}\cup\Sigma_{o21}^{min}|$. Fortunately, some NP-complete minimization problems have fairly accurate polynomial time approximation algorithms. This paper further discuss approximation methods that can be used for Problem 1.

For a thorough discussion of approximation algorithms, see Ausiello et al. (1999), but to better quantify what is meant by an approximation to Problem 1, suppose P is the set of instances of Problem 1. Let $p\in P$ be a specific problem instance corresponding to G, H, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}. Suppose $(\Sigma_{o12}^{min}(p),\Sigma_{o21}^{min}(p))$ is the solution of this problem instance and A is an algorithm that when given input p, returns $(\Sigma_{o12}^{A}(p),\Sigma_{o21}^{A}(p))$ such that $\mathcal{L}(H)$ co-observable with respect to $\mathcal{L}(G)$, $\Sigma_{o1}\cup\Sigma_{o21}^{A}(p)$, $\Sigma_{o2}\cup\Sigma_{o12}^{A}(p)$, Σ_{c1} and Σ_{c2}. The closeness of the approximation $(\Sigma_{o12}^{A}(p),\Sigma_{o21}^{A}(p))$ is measured with the ratio

$$\frac{|\Sigma_{o12}^{min}(p)\cup\Sigma_{o21}^{min}(p)|}{|\Sigma_{o12}^{A}(p)\cup\Sigma_{o21}^{A}(p)|}. \qquad (1)$$

3. THE GRAPH CUTTING AND SENSOR SELECTION PROBLEMS

It is now shown how the communication selection problem is related to a special type of sensor selection problem and a type of directed graph st-cut problem. Examples of constructions in this section are in Rohloff and van Schuppen (2004).

For the sensor selection problem, a control designer may be given a choice of what events the controller may observe. The set $\Sigma_o\subseteq\Sigma$ is a *sufficient sensor selection* with respect to G, H and Σ_c if $\mathcal{L}(H)$ is observable with respect to $\mathcal{L}(G)$, Σ_o and Σ_c. The problem of finding a minimal cardinality set of observable events is called the *sensor selection problem.*

Problem 3. Sensor Selection: Given G, H and Σ_c, find a sufficient sensor selection Σ_o^{min} such that for any other sufficient sensor selection Σ_o, $|\Sigma_o^{min}|\leq|\Sigma_o|$.

Note that a solution to Problem 3 exists because Σ_o is a finite set, but in general it is not unique.

For the graph cutting problem, suppose an edge-colored directed graph $D=(V,A,C)$ is given where V is a set of vertices, $A\subseteq V\times V$ are directed edges, $C=\{c_1,\ldots,c_p\}$ is the set of colors and for $s,t\in V$, there is a path of directed edges from s to t. Each edge is assigned a color in C and let A_i be the edges having color c_i. Given $I\subseteq C$, let $A_I=\cup_{c_i\in I}A_i$. The set I is a colored st-cut if $(V,A\setminus A_I,C)$ has no path from s to t. This prompts the definition of the colored cut problem.

Problem 4. Minimal Colored Cut: Given an edge colored directed graph $D=(V,A,C)$ and two vertices, $s,t\in V$, find a colored st-cut $I^{min}\subseteq C$ such that for any other colored st-cut $I\subseteq C$, $|I^{min}|\leq|I|$.

Note that an automaton can be represented as a colored directed graph where transition labelling can be thought of as colorings. From Khuller et al. (2004), instances of Problem 3 can be converted to instances of Problem 4 and vice-versa while preserving approximation properties. Due to Khuller et al. (2004), solutions to these problems are difficult to approximate.

Theorem 5. (Khuller et al. (2004)) Problem 3 and Problem 4 admit no $2^{\log^{(1-\epsilon)}n}$ approximation for any $\epsilon>0$ unless $NP\subseteq DTIME(n^{\text{polylog }n})$.

This lower bound is generally considered to be a very poor approximation result because as $\epsilon\to 0$, then $2^{\log^{(1-\epsilon)}n}\to n$. Furthermore, it is believed that $NP\not\subseteq DTIME(n^{\text{polylog }n})$ (Arora and Lund (1997)). Due to reductions between these problems, similar properties are shown below for Problem 1.

3.1 Graph Cutting for Communication Selection

A nondeterministic automaton construction is given by Rudie and Willems (1995) to test if $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}. This subsection presents a modified version of this construction to convert an instance of Problem 1 into an instance of Problem 4.

Suppose G, H, Σ_{o1}, Σ_{o2}, Σ_{o12}, Σ_{o21}, Σ_{c1} and Σ_{c2} are given. A nondeterministic automaton $\mathcal{M}_{\Sigma_{o12}\Sigma_{o21}}$ can be constructed to test if $(\Sigma_{o12},\Sigma_{o21})$ is a sufficient communication selection.

Let Σ_1 and Σ_2 be disjoint sets of events such that for all $i\in\{1,2\}$, $\Sigma_i\cap\Sigma=\emptyset$. Furthermore, define $\Psi_i:\Sigma\to\Sigma_i$ for $i\in\{1,2\}$ to be a one-to-one function, and for $\sigma\in\Sigma$, $\Psi_i(\sigma)$ is called σ_i when it can be done without ambiguity. The automaton $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}=(X^{\Sigma_{o12},\Sigma_{o21}},x_0^{\Sigma_{o12},\Sigma_{o21}},(\Sigma\cup\Sigma_1\cup\Sigma_2),\delta^{\Sigma_{o12},\Sigma_{o21}},X_m^{\Sigma_{o12},\Sigma_{o21}})$ can then be defined where $X^{\Sigma_{o12},\Sigma_{o21}}=X^H\times X^H\times X^H\times G^G\cup\{d\}$, $x_0^{\Sigma_{o12},\Sigma_{o21}}=(x_0^H,x_0^H,x_0^H,x_0^G)$. The notation is used that $x \overset{\gamma}{\mapsto}_{\Sigma_{o12},\Sigma_{o21}} y$ represents that there is a transition according to the transition rules of $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ from state x to state y labelled by event γ.

The transition structure of $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ is nondeterministic so for a state, $x \in X^{\Sigma_{o12},\Sigma_{o21}}$ and an event $\gamma \in \Sigma \cup \Sigma_1 \cup \Sigma_2$, $\delta^{\Sigma_{o12},\Sigma_{o21}}(x,\gamma)$ can be a set of states as is represented below. Therefore, $y \in \delta^{\Sigma_{o12},\Sigma_{o21}}(x,\gamma)$ if an only if $x \overset{\gamma}{\mapsto}_{\Sigma_{o12},\Sigma_{o21}} y$. The state transition representations are also extended in the usual manner to be defined over strings of transitions.

In the formal definition of the transition relation, the $(*)$ condition holds at a state $x = (x_1,x_2,x_3,x_4)$ if

$$\left.\begin{array}{l} \delta^H(x_1,\sigma) \text{ is defined if } \sigma \in \Sigma_{c1} \\ \delta^H(x_2,\sigma) \text{ is defined if } \sigma \in \Sigma_{c2} \\ \delta^H(x_3,\sigma) \text{ is not defined} \\ \delta^G(x_4,\sigma) \text{ is defined} \end{array}\right\}. \qquad (*)$$

The transition relation for $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ is defined such that $d \in \delta^{\Sigma_{o12},\Sigma_{o21}}((x_1,x_2,x_3,x_4),\sigma)$ if $(*)$ and $(\delta^H(x_1,\sigma),\delta^H(x_2,\sigma),\delta^H(x_3,\sigma),\delta^G(x_4,\sigma)) \in \delta^{\Sigma_{o12},\Sigma_{o21}}((x_1,x_2,x_3,x_4),\sigma)$ for all $\sigma \in \Sigma$. In addition, for $\sigma \in \Sigma \setminus (\Sigma_{o1} \cup \Sigma_{o2})$,

$$\delta^{\Sigma_{o12},\Sigma_{o21}}((x_1,x_2,x_3,x_4),\sigma) \subseteq \left\{\begin{array}{l} (\delta^H(x_1,\sigma),x_2,x_3,x_4) \\ (x_1,\delta^H(x_2,\sigma),x_3,x_4) \\ (x_1,x_2,\delta^H(x_3,\sigma),\delta^G(x_4,\sigma)) \end{array}\right\}.$$

For $\sigma \in \Sigma_{o2} \setminus (\Sigma_{o1} \cup \Sigma_{o21})$,

$$\delta^{\Sigma_{o12},\Sigma_{o21}}((x_1,x_2,x_3,x_4),\Psi_1(\sigma)) \subseteq \left\{\begin{array}{l} (\delta^H(x_1,\sigma),x_2,x_3,x_4) \\ (x_1,\delta^H(x_2,\sigma),\delta^H(x_3,\sigma),\delta^G(x_4,\sigma)) \end{array}\right\}.$$

For $\sigma \in \Sigma_{o1} \setminus (\Sigma_{o2} \cup \Sigma_{o12})$,

$$\delta^{\Sigma_{o12},\Sigma_{o21}}((x_1,x_2,x_3,x_4),\Psi_2(\sigma)) \subseteq \left\{\begin{array}{l} (x_1,\delta^H(x_2,\sigma),x_3,x_4) \\ (\delta^H(x_1,\sigma),x_2,\delta^H(x_3,\sigma),\delta^G(x_4,\sigma)) \end{array}\right\}.$$

No other transitions are defined in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$.

The construction for $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ is modified from the construction presented by Rudie and Willems (1995) in that $\Psi_1(\sigma)$ and $\Psi_2(\sigma)$ transitions correspond to state estimation updates that could be removed if σ observances would be communicated between the controllers. The $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ construction prompts the following corollary to the main result of Rudie and Willems (1995).

Corollary 6. The state d is reachable from the initial state in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ if and only if $\mathcal{L}(H)$ is not co-observable with respect to $\mathcal{L}(G)$, $(\Sigma_{o1} \cup \Sigma_{o21})$, $(\Sigma_{o2} \cup \Sigma_{o12})$, Σ_{c1} and Σ_{c2}.

Note that $\mathcal{M}_{(\Sigma_{o12}\cup\{\sigma\}),\Sigma_{o21}}$ can be constructed from $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ by cutting all transitions labelled by $\Psi_2(\sigma)$. Therefore, the act of controller 1 communicating all occurrences of event σ to controller 2 corresponds to trimming all $\Psi_2(\sigma)$ labelled transitions in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$. Similar properties hold for the adding a σ event to Σ_{o21} and respectively trimming $\Psi_1(\sigma)$ labelled transitions in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$.

Define $\Sigma_j^{oij} = \{\Psi_j(\sigma)|\sigma \in \Sigma^{oij}\}$. A set of events $\Sigma_1^{o21} \cup \Sigma_2^{o12}$ is a $x_0^{\emptyset,\emptyset}$ d-cut in $\mathcal{M}_{\emptyset,\emptyset}$ if and only if d is not reachable in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$. Therefore, the pair $(\Sigma_{o12}^{min},\Sigma_{o21}^{min})$ is the smallest cardinality communication selection if and only if the corresponding events $\Sigma_1^{o21min} \cup \Sigma_2^{o12min} \subseteq \Sigma_1 \cup \Sigma_2$ is the smallest cardinality $x_0^{\emptyset,\emptyset}$ d-cut in $\mathcal{M}_{\emptyset,\emptyset}$ when restricted to cutting transitions labelled with events in $\Sigma_1 \cup \Sigma_2$. This realization effectively converts the communication selection problem into a restricted form of Problem 4. Similar constructions also exist for the case of more than two controllers. A construction is now shown to convert the restricted graph cutting problem into a true instance of Problem 4. First define:

$$X_x^{\Sigma_{o12},\Sigma_{o21}} = \left\{y | \exists t \in \Sigma^*, x \overset{t}{\mapsto}_{\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}} y\right\}.$$

$X_x^{\Sigma_{o12},\Sigma_{o21}}$ represents all states that could be reached from x in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ if only Σ transitions were allowed. The states in $X_x^{\Sigma_{o12},\Sigma_{o21}}$ would be reachable from x in $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$ no matter what events are communicated between the controllers. With this in mind, an automaton $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}}$ is constructed from $\mathcal{M}_{\Sigma_{o12},\Sigma_{o21}}$. It is known that $d \notin X_{x_0}^{\Sigma_{o12},\Sigma_{o21}}$. Let $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}} = (\tilde{X}^{\Sigma_{o12},\Sigma_{o21}}, \tilde{x}_0^{\Sigma_{o12},\Sigma_{o21}}, \Sigma_1 \cup \Sigma_2, \tilde{\delta}^{\Sigma_{o12},\Sigma_{o21}})$, where $\tilde{X}^{\Sigma_{o12},\Sigma_{o21}} := X^H \times X^H \times X^H \times X^G \cup \{d\}$ and $\tilde{x}_0^{\Sigma_{o12},\Sigma_{o21}} := (x_0^H, x_0^H, x_0^H, x_0^G)$.

The transition relation $\tilde{\delta}^{\Sigma_{o12},\Sigma_{o21}}$ is defined as follows. Suppose there exists three states $x,y,z \in X^{\Sigma_{o12},\Sigma_{o21}}$ and $\sigma \in \Sigma$ such that $z \in X_x^{\Sigma_{o12},\Sigma_{o21}}$, $z \overset{\sigma_i}{\mapsto}_{\Sigma_{o12},\Sigma_{o21}} y$ where $\sigma_i \in \Sigma_1 \cup \Sigma_2$. Then,

$$\tilde{\delta}^{\Sigma_{o12},\Sigma_{o21}}(x,\sigma_i) = \left\{\begin{array}{l} y \text{ if } d \notin X_y^{\Sigma_{o12},\Sigma_{o21}} \\ d \text{ if } d \in X_y^{\Sigma_{o12},\Sigma_{o21}} \end{array}\right\}.$$

This construction prompts the below theorem.

Theorem 7. Given an $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$ as constructed above, $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, $(\Sigma_{o1} \cup \Sigma_{o21}), (\Sigma_{o2} \cup \Sigma_{o12})$ and Σ_{c1}, Σ_{c2} if and only if $\Sigma_1^{o21} \cup \Sigma_2^{o12}$ is a colored $\tilde{x}_0^{\emptyset,\emptyset}$ d-cut in the colored directed graph $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$.

Note that H, G, Σ_{o1}, Σ_{o2}, Σ_{c1} and Σ_{c2}, $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$ can be constructed in polynomial time. This prompts the following corollary due to the stated approximation difficulty results for Problem 4

Corollary 8. The communication selection problem admits no $2^{\log^{(1-\epsilon)} n}$ approximation for any $\epsilon > 0$ unless $NP \subseteq DTIME(n^{\text{polylog } n})$.

Therefore, solutions to the communication selection problem are very difficult to approximate.

4. HEURISTIC APPROXIMATION METHODS

Heuristic algorithms are now shown to approximate solutions to the communication selection problem. These algorithms are based on graph cuttings of $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$. After constructing $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$, events are iteratively assigned to be communicated between the controllers in order to cut all paths from $\tilde{x}_o^{\emptyset,\emptyset}$ to d in $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$. The first algorithm, called *DetGreedyAprx*, is a deterministic greedy algorithm that uses a utility function to identify and iteratively cut transitions associated with event labels in $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}}$.

4.1 A Deterministic Greedy Algorithm

Starting with a trim version of $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$, suppose it is desirable to find the "probability" $\mathcal{P}(\sigma, \tilde{\mathcal{M}}_{\emptyset,\emptyset})$ that a "randomly" selected path from $\tilde{x}_o^{\emptyset,\emptyset}$ to d contains an edge labelled by $\Psi_1(\sigma) \in \Sigma_1$ or $\Psi_2(\sigma) \in \Sigma_2$ corresponding to $\sigma \in \Sigma$. By selecting an event which occurs with a relatively high probability on paths from $\tilde{x}_o^{\emptyset,\emptyset}$ to d, then that event should have a high utility of being communicated between the controllers. The term "probability" and "randomly" are used here in a loose and intuitive manner in order to develop an understanding for the solution method for this problem while avoiding the explicit definition of a probability distribution function at this time.

In order to remove all paths to d in $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$, it would be desirable to first cut transitions associated with events with the highest utility $\mathcal{P}(\sigma, \tilde{\mathcal{M}}_{\emptyset,\emptyset})$. After an event is selected to be cut in $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$, the utility function $\mathcal{P}(\cdot,\cdot)$ is updated to reflect the changes in the communication sets and another event is then chosen to be communicated. This procedure, seen in Algorithm DetGreedyAprx, is iterated until there are no paths to d.

As DetGreedyAprx iteratively chooses events to communicate between controllers, the $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$ automaton is continually trimmed. Therefore, as Σ_{o12} and Σ_{o21} are updated, the next $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$ can be calculated in polynomial time. The relative probabilities $\{\rho^1, \ldots, \rho^k\}$ associated with the

Deterministic Greedy Approximation Algorithm (DetGreedyAprx)

Input: $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$;
$\Sigma_{o12} \leftarrow \emptyset$, $\Sigma_{o21} \leftarrow \emptyset$, $i \leftarrow 1$;
$\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}} \leftarrow Trim(\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}})$;
While d reachable in $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$;
{
$\sigma^i \leftarrow \arg\max_{\sigma \in (\Sigma_{o1} \cup \Sigma_{o2})} \left(\mathcal{P}\left(\sigma, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}\right)\right)$;
$\rho^i \leftarrow \mathcal{P}\left(\sigma^i, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}\right)$;
$\Sigma_{o12} \leftarrow \Sigma_{o12} \cup (\{\sigma^i\} \cap \Sigma_{o1})$;
$\Sigma_{o21} \leftarrow \Sigma_{o21} \cup (\{\sigma^i\} \cap \Sigma_{o2})$;
$k \leftarrow i$;
$i \leftarrow i + 1$;
Reconstruct $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}}$;
$\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}} \leftarrow Trim(\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}})$;
}
Return $\Sigma_{o12}, \Sigma_{o21}$;

events selected for communication, $\{\sigma^1, \ldots, \sigma^k\}$, are stored for later analysis of the accuracy of the found approximation $|\Sigma_{o12} \cup \Sigma_{o21}|$.

It remains to be discussed how $\mathcal{P}\left(\sigma, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}\right)$ is calculated. At each state x in $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$, suppose there are κ_x output transitions. $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$ is converted into a stochastic automaton by assigning a probability of occurrence $\frac{1}{\kappa_x}$ to each output transition of x. Let $\mathcal{P}(\sigma, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}})$ denote the probability that a random walk in $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$ traverses a σ transition from the initial state to d. It should be noted that this probability can be computed in polynomial time using standard methods from Hoel et al. (1971). Therefore, this approximation algorithm runs in polynomial time.

Algorithm DetGreedyAprx is now analyzed to obtain a bounds on accuracy of the approximation returned by the algorithm. The sets $\Sigma^{\min_i}_{o12}$ and $\Sigma^{\min_i}_{o21}$ denote the minimum cardinality communication selection sets that could be chosen at iteration i given that events in Σ^i_{o12} and Σ^i_{o21} are already selected to be communicated. Naturally, $\Sigma^{\min_1}_{o12} = \Sigma^{\min}_{o12}$ and $\Sigma^{\min_1}_{o21} = \Sigma^{\min}_{o21}$.

Lemma 9. In DetGreedyAprx, on the ith iteration,

$$\frac{1}{\mathcal{P}\left(\sigma^i, \tilde{\mathcal{M}}_{\Sigma^i_{o12},\Sigma^i_{o21}}\right)} \leq |\Sigma^{\min_i}_{o12} \cup \Sigma^{\min_i}_{o21}|$$

Lemma 9 can be used to show the following result on the closeness of the approximation returned by DetGreedyAprx.

Theorem 10. For the sets $\Sigma_{o12}, \Sigma_{o21}$ returned by DetGreedyAprx and a minimum communication set $\Sigma^{\min}_{o12}$, $\Sigma^{\min}_{o21}$,

$$\frac{|\Sigma_{o12} \cup \Sigma_{o21}|}{|\Sigma_{o12}^{\min} \cup \Sigma_{o21}^{\min}|} \leq \sum_{i=1}^{|\Sigma_{o12} \cup \Sigma_{o21}|} \rho_i$$

where $\{\rho^1, \dots \rho^k\}$ are the iterative probabilities stored during the operation of DetGreedyAprx.

Because of Theorem 10, a bound on the closeness of the approximation returned by DetGreedyAprx can be calculated. Unfortunately $\sum_{i=1}^{k} \rho^i$ can be on the order of $n - \epsilon$ in the worst case where n is the number of system events and ϵ is some constant greater than 0. A lower bound on the closeness of the bound on the approximation ratio shown in Theorem 10 is now shown.

Theorem 11. From a set $\{\rho^1, \dots, \rho^k\}$ calculated from a running of DetGreedyAprx,

$$\sum_{i=1}^{k} \rho^i \geq H_k = \sum_{j=1}^{k} \frac{1}{j}.$$

Although Theorem 11 puts a lower bound on the guarantee of the approximation ratio shown in Theorem 10, DetGreedyAprx may return a solution with an approximation ratio better than H_k.

4.2 A Randomized Greedy Algorithm

A randomized greedy approximation algorithm, *RandGreedyAprx*, is now given based on the DetGreedyAprx method of iteratively cutting the $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}}$ automaton. This algorithm randomly enables events to be communicated but uses the utility function $\mathcal{P}(\sigma, \tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}})$ to weight the probability distribution of a sensor being selected. Therefore, an event with a relatively high probability of occurring over the set of all paths to d in $\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}}$ will have a higher probability of being added to $\Sigma_{o12}, \Sigma_{o21}$.

Note that most likely RandGreedyAprx returns a different solution every time it is run. Therefore, RandGreedyAprx can be iterated multiple times to boost the probability that a good approximate solution will be found.

5. DISCUSSION

This paper has shown connections between a communicating controller problem and a special graph cutting problem. This shows that solutions to the communicating controller problem are difficult to approximate in the worst case. However, the connection allows for the design of several heuristic algorithms that could work well in practice for the communicating controller problem.

Randomized Greedy Approximation Algorithm (RandGreedyAprx):
Input: $\tilde{\mathcal{M}}_{\emptyset,\emptyset}$;
$\Sigma_{o12} \leftarrow \emptyset$, $\Sigma_{o21} \leftarrow \emptyset$, $i \leftarrow 1$;
$\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}} \leftarrow Trim(\tilde{\mathcal{M}}_{\Sigma_{o12},\Sigma_{o21}})$;
While d reachable in $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$;
{
$\Sigma_s \leftarrow (\Sigma_{o1} \cup \Sigma_{o2}) \setminus (\Sigma_{o12} \cup \Sigma_{o21})$
For all $\sigma \in \Sigma_s$
{
$\Pr(\sigma) \leftarrow \frac{\mathcal{P}(\sigma, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}})}{\sum_{\gamma \in \Sigma_s} \left(\mathcal{P}\left(\gamma, \tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}\right)\right)}$;
}
Randomly select $\sigma_i \in \Sigma_s$ according to probability distribution $\Pr(\sigma)$;
$k \leftarrow i$;
Remove σ_i labelled transitions in $\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}}$;
$\Sigma_{o12} \leftarrow \Sigma_{o12} \cup (\{\sigma^i\} \cap \Sigma_{o1})$;
$\Sigma_{o21} \leftarrow \Sigma_{o21} \cup (\{\sigma^i\} \cap \Sigma_{o2})$;
$i \leftarrow i + 1$;
$\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}} \leftarrow Trim(\tilde{\mathcal{M}}^T_{\Sigma_{o12},\Sigma_{o21}})$;
}
Return $\Sigma_{o12}, \Sigma_{o21}$;

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